

Doubly heavy Baryons from QCD Spectral Sum Rules

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Abstract

We consider the ratios of doubly heavy baryon masses using Double Ratios of Sum Rules (DRSR), which are more accurate than the usual simple ratios used for getting hadron masses. Our results are comparable with the ones from potential models. In our approach, the α_s corrections induced by the anomalous dimensions of the correlators are the main sources of the $\Xi_{QQ}^* - \Xi_{QQ}$ mass-splittings, which seem to indicate a $1/M_Q$ behaviour and can only allow the electromagnetic decay $\Xi_{QQ}^* \rightarrow \Xi_{QQ} + \gamma$ but not to $\Xi_{QQ} + \pi$. Our results also show that the SU(3) mass-splittings are (almost) independent of the spin of the baryons and behave approximately like $1/M_Q$, which could be understood from the QCD expressions of the corresponding two-point correlator. Our results can be improved by including radiative corrections to the SU(3) breaking terms and can be tested, in the near future, at Tevatron and LHCb.

Keywords: QCD spectral sum rules, baryon spectroscopy, heavy quarks.

1. Introduction

In a previous paper [1], we have considered, using Double Ratios (DRSR) [2] of QCD spectral sum rules (QSSR) [3, 4], the splittings due to SU(3) breakings of the baryons made with one heavy quark. This project has been pursued in the case of doubly heavy baryons in [5], which will be reviewed in this talk.

The absolute values of the doubly heavy baryon masses of spin 1/2 ($\Xi_{QQ} \equiv QQq$) and spin 3/2 ($\Xi_{QQ}^* \equiv QQq$) have been obtained using QCD spectral sum rules (QSSR) (for the first time) in [6] with the results in GeV:

$$\begin{aligned} M_{\Xi_{cc}^*} &= 3.58(5) \quad , \quad M_{\Xi_{bb}^*} = 10.33(1.09) \, , \\ M_{\Xi_{cc}} &= 3.48(6) \quad , \quad M_{\Xi_{bb}} = 9.94(91) \, , \end{aligned} \quad (1)$$

and in [7]:

$$M_{\Xi_{bcu}} = 6.86(28) \, . \quad (2)$$

More recently [8, 9], some results have been obtained using some particular choices of the interpolating currents. The predictions for $M_{\Xi_{cc}^*}$ and $M_{\Xi_{cc}}$ are in good

agreement with the experimental candidate $M_{\Xi_{cc}} = 3518.9$ MeV [10]. In the following, we shall improve these previous predictions using the DRSR for estimating the ratio of the 3/2 over the 1/2 baryon masses as well as their splittings due to SU(3) breakings, which we shall compare with some potential model predictions [7, 11–13].

2. The Interpolating Currents

For the spin 1/2 (QQq) baryons, and following Ref. [6], we work with the lowest dimension currents:

$$J_{\Xi_Q} = \epsilon_{\alpha\beta\lambda} \left[(Q_\alpha^T C \gamma_5 q_\beta) + b (Q_\alpha^T C q_\beta) \gamma_5 \right] Q_\lambda, \quad (3)$$

where $q \equiv d, s$ are light quark fields, $Q \equiv c, b$ are heavy quark fields, b is *a priori* an arbitrary mixing parameter. Its value has been found to be: $b = -1/5$, in the case of light baryons [14] and in the range [1, 15, 16]:

$$-0.5 \leq b \leq 0.5 \, , \quad (4)$$

for non-strange heavy baryons. The corresponding two-point correlator reads:

$$\begin{aligned} S(q) &= i \int d^4x e^{iqx} \langle 0 | \mathcal{T} \bar{J}_{\Xi_Q}(x) J_{\Xi_Q}(0) | 0 \rangle \\ &\equiv \hat{q} F_1 + F_2 \, , \end{aligned} \quad (5)$$

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where F_1 and F_2 are two invariant functions.

For the spin 3/2 (QQQ) baryons, we also follow Ref. [6] and work with the interpolating currents:

$$J_{\Xi_Q}^\mu = \sqrt{\frac{1}{3}} \epsilon_{\alpha\beta\lambda} [2(Q_\alpha^T C \gamma^\mu d_\beta) Q_\lambda + (Q_\alpha^T C \gamma^\mu Q_\beta) q_\lambda] \quad (6)$$

The corresponding two-point correlator reads:

$$\begin{aligned} S^{\mu\nu}(q) &= i \int d^4x e^{iqx} \langle 0 | \mathcal{T} \bar{J}_{\Xi_Q}^\mu(x) J_{\Xi_Q}^\nu(0) | 0 \rangle \\ &\equiv g^{\mu\nu} (\hat{q} F_1 + F_2) + \dots \end{aligned} \quad (7)$$

3. The Two-Point Correlator in QCD

The expressions of the two-point correlator using the previous interpolating currents have been obtained in the chiral limit $m_q = 0$ and including the mixed condensate contributions by [6]. In this work, we extend these results by including the linear strange quark mass corrections to the perturbative and $\langle \bar{s}s \rangle$ condensate contributions. The explicit expressions can be found in Ref. [5].

4. Form of the Sum Rules

We parametrize the spectral function using the standard duality ansatz: “one resonance”+ “QCD continuum”. The QCD continuum starts from a threshold t_c and comes from the discontinuity of the QCD diagrams. Transferring its contribution to the QCD side of the sum rule, one obtains the finite energy inverse Laplace sum rules [3, 17, 18]. Consistently, we also take into account the SU(3) breaking at the continuum threshold t_c :

$$\sqrt{t_c}|_{SU(3)} \simeq \left(\sqrt{t_c}|_{SU(2)} \equiv \sqrt{t_c} \right) + \bar{m}_s, \quad (8)$$

where \bar{m}_s is the running strange quark mass. As we do an expansion in m_s , we take the threshold $t_q = 4m_Q^2$ for consistency, where m_Q is the heavy quark mass, which we shall take in the range covered by the running and on-shell mass because of its ambiguous definition when working to lowest order (LO). As usually done in the sum rule literature, one can estimate the baryon masses from the following ratios ($i = 1, 2$):

$$\mathcal{R}_i^q = \frac{\int_{t_q}^{t_c} dt t e^{-t\tau} \text{Im} F_i(t)}{\int_{t_q}^{t_c} dt e^{-t\tau} \text{Im} F_i(t)}, \quad \mathcal{R}_{21}^q = \frac{\int_{t_q}^{t_c} dt e^{-t\tau} \text{Im} F_2(t)}{\int_{t_q}^{t_c} dt e^{-t\tau} \text{Im} F_1(t)} \quad (9)$$

where at the τ -stability point :

$$M_{B_q^{(*)}} \simeq \sqrt{\mathcal{R}_i^q} \simeq \mathcal{R}_{21}^q, \quad (i = 1, 2). \quad (10)$$

These predictions lead to a typical uncertainty of 10-15% [6, 7, 16], which are not competitive compared

with predictions from some other approaches, especially from potential models [7, 11]. In order to improve the QSSR predictions, we work with the double ratio of sum rules (DRSR):

$$r_i^{sd} \equiv \sqrt{\frac{\mathcal{R}_i^s}{\mathcal{R}_i^d}}, \quad (i = 1, 2); \quad r_{21}^{sd} \equiv \frac{\mathcal{R}_{21}^s}{\mathcal{R}_{21}^d}, \quad (11)$$

which take directly into account the SU(3) breaking effects. These quantities are obviously less sensitive to the choice of the heavy quark masses and to the value of the t_c than the simple ratios \mathcal{R}_i and \mathcal{R}_{21} .

5. The Ξ_{QQ}^*/Ξ_{QQ} mass ratio

We extract the mass ratios using the DRSR analogue of the one in Eq. (11) which we denote by:

$$r_i^{3/1} \equiv \sqrt{\frac{\mathcal{R}_i^3}{\mathcal{R}_i^1}}, \quad (i = 1, 2); \quad r_{21}^{3/1} \equiv \frac{\mathcal{R}_{21}^3}{\mathcal{R}_{21}^1}, \quad (12)$$

where the upper indices 3 and 1 correspond respectively to the spin 3/2 and 1/2 channels. We use the QCD expressions of the two-point correlators given by [6] which we have checked. In our analysis, we truncate the QCD series at the dimension 4 condensates until which we have calculated the m_s corrections. We shall only include the effect of the mixed condensate (if necessary) for controlling the accuracy of the approach or for improving the τ or/and t_c stability of the analysis.

The charm quark channel

Fixing $t_c = 25 \text{ GeV}^2$ and $\tau = 0.8 \text{ GeV}^{-2}$, which are inside the t_c and τ -stability regions (see Fig. 2a and Fig. 2b), we show in Fig. 1 the b -behaviour of $r^{3/1}$ which shows that $r_1^{3/1}$ and $r_2^{3/1}$ are very stable but not $r_{12}^{3/1}$. We then eliminate $r_{12}^{3/1}$, where one can notice some common solutions for:

$$b \simeq -0.35, \quad \text{and} \quad b \simeq +0.2, \quad (13)$$

which are inside the range given in Eq. (4). For definiteness, we fix $b = -0.35$ and study the τ -dependence of the result in Fig. 2a and its t_c -dependence in Fig. 2b. The large stability in t_c confirms our expectation for the weak t_c -dependence of the DRSR. In these figures, we have used $m_c = 1.26 \text{ GeV}$ and have checked that the results are insensitive to the change of mass to $m_c = 1.47 \text{ GeV}$. We have also checked that the inclusion of the mixed condensate contribution does not affect the present result (within the high-accuracy obtained here) obtained by retaining the dimension-4 condensates.

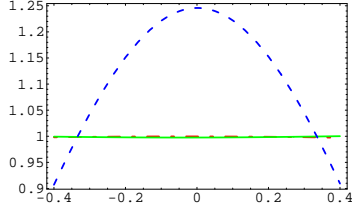


Figure 1: **Charm quark** : b -behaviour of the different DRSR given $\tau = 0.8 \text{ GeV}^{-2}$ and $t_c = 25 \text{ GeV}^2$. $r_1^{3/1}$ dot-dashed line (red); $r_2^{3/1}$ continuous line (green); $r_{12}^{3/1}$ dashed line (blue). We have used $m_c = 1.26 \text{ GeV}$ and the other QCD parameters in ref. [5].

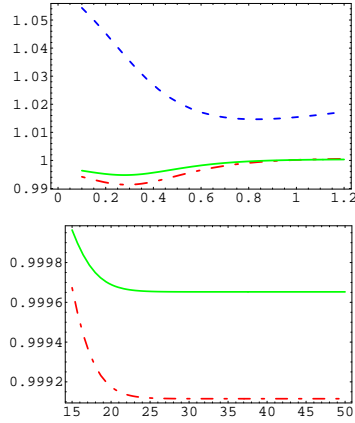


Figure 2: Ξ_{cc}^*/Ξ_{cc} a) τ -behaviour of $r_1^{3/1}$: dot-dashed line (red) and $r_2^{3/1}$: continuous line (green) with $b = -0.35$ and $t_c = 25 \text{ GeV}^2$. b) t_c -behaviour of $r_1^{3/1}$: dot-dashed line (red) and $r_2^{3/1}$: continuous line (green) with $b = -0.35$ and $\tau = 0.8 \text{ GeV}^{-2}$.

Results of the analysis

From the analysis of the charm and bottom quark channels, we deduce with high accuracy to lowest order: $M_{\Xi_{cc}^*}/M_{\Xi_{cc}} = 0.9994(3)$ and $M_{\Xi_{bb}^*}/M_{\Xi_{bb}} = 1.0000$, while the inclusion of the radiative corrections induced by the anomalous dimension of the correlators modifies the previous results to (see Table 1):

$$\frac{M_{\Xi_{cc}^*}}{M_{\Xi_{cc}}} = 1.0167(19), \quad \frac{M_{\Xi_{bb}^*}}{M_{\Xi_{bb}}} = 1.0019(3), \quad (14)$$

which would correspond to the mass-splittings in MeV:

$$M_{\Xi_{cc}^*} - M_{\Xi_{cc}} = 59(7), \quad M_{\Xi_{bb}^*} - M_{\Xi_{bb}} = 19(3), \quad (15)$$

comparable with standard potential models [7, 11] but not with the one of about 24 MeV obtained in [13] for the charm (see Table 1). Our result excludes the possibility that $M_{\Xi_{QQ}^*} \geq M_{\Xi_Q} + m_\pi$, indicating that the Ξ_{QQ}^* can only decay electromagnetically but not to $\Xi_Q + \pi$.

6. The Ω_{QQ}/Ξ_{QQ} mass ratio

We use the DRSR in Eq. (11) where their QCD expressions can be obtained from the one of the two-point

correlator in [6, 7], while the new quark mass corrections can be found in [5]. The analysis for the charm quark is shown in Fig. 3, from which we can deduce the result given in Table 1. A similar analysis for the bot-

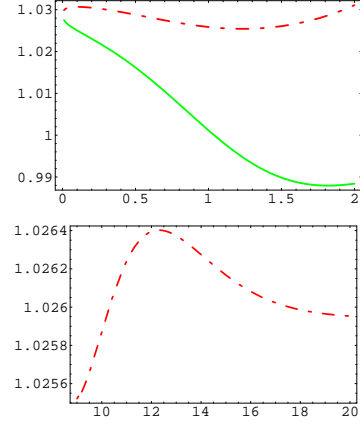


Figure 3: Ω_{cc}^*/Ξ_{cc} a) τ -behaviour of $r_2^{sd}(cc)$: continuous line (green) and $r_1^{sd}(cc)$: dot-dashed line (red) in the charm quark channel for $b = -0.35$, $t_c = 12 \text{ GeV}^2$ and $m_c = 1.26 \text{ GeV}$. b) t_c -behaviour of $r_1^{sd}(cc)$ for $\tau = 1 \text{ GeV}^{-2}$: dot-dashed line (red)

tom quark is also given in Table 1. We deduce from the ratios (in units of MeV):

$$M_{\Omega_{cc}^*} - M_{\Xi_{cc}^*} = 92(24), \quad M_{\Omega_{bb}^*} - M_{\Xi_{bb}^*} \simeq 49(13). \quad (16)$$

Our results indicate an approximate decrease like $1/m_Q$ of the mass splittings from the c to the b quark channels. This behaviour can be qualitatively understood from the QCD expressions of the corresponding correlator, where the m_s corrections enter like m_s/m_Q , and which can be checked using alternative methods.

7. The Ω_{QQ}^*/Ξ_{QQ}^* mass ratio

We pursue our analysis for the spin 3/2 baryons. We deduce, for the charm quark, at the stability regions, the ratios given in Table 1 and the corresponding mass-splittings (in units of MeV):

$$M_{\Omega_{cc}^*} - M_{\Xi_{cc}^*} = 94(27), \quad M_{\Omega_{bb}^*} - M_{\Xi_{bb}^*} = 50(15), \quad (17)$$

which agree with the potential model results given in [7] (see Table 1). Again, like in the case of spin 1/2 baryons, the SU(3) mass-differences appear to behave like $1/M_Q$, which can be inspected from the QCD expressions of the two-point correlator.

One can also observe that the mass-splittings are almost the same for the spin 1/2 and 3/2 baryons.

8. The Ω_{bc}/Ξ_{bc} mass ratio

The Ξ_{bc} and the Ω_{bc} spin 1/2 baryons can be described by the corresponding currents [6, 7]:

$$\begin{aligned} J_{\Lambda_{bc}} &= \epsilon_{\alpha\beta\lambda} \left[(c_\alpha^T C \gamma_5 d_\beta) + k(c_\alpha^T C d_\beta) \gamma_5 \right] b_\lambda, \\ J_{\Omega_{bc}} &= J_{\Xi_{bc}} \quad (d \rightarrow s), \end{aligned} \quad (18)$$

where d, s are light quark fields, c, b are heavy quark fields and k is *a priori* an arbitrary mixing parameter. Like in previous sections, we study the different ratio of moments for this case. The b -stability is obtained for $k \pm 0.05$ while the τ and t_c behaviours are also very stable at which we deduce the DRSR in Table 1 and the corresponding splitting:

$$M_{\Omega_{bc}} - M_{\Xi_{bc}} = 41(7) \text{ MeV}. \quad (19)$$

Table 1: QSSR predictions for the doubly heavy baryons mass ratios and splittings, which we compare with the Potential Model (PM) range of results in [7, 12]. The PM prediction for the spin 3/2 is an average with the one for spin 1/2. The mass inputs are in GeV and the mass-splittings are in MeV.

Mass ratios	Mass inputs	Mass plittings	PM
$\Xi_{cc}^*/\Xi_{cc} = 1.0167(19)$	$\Xi_{cc} = 3.52[10]$	$\Xi_{cc}^* - \Xi_{cc} = 59(7)$	70-93
$\Xi_{bb}^*/\Xi_{bb} = 1.0019(3)$	$\Xi_{bb} = 9.94[6]$	$\Xi_{bb}^* - \Xi_{bb} = 19(3)$	30-38
$\Omega_{cc}/\Xi_{cc} = 1.0260(70)$	$\Xi_{cc} = 3.52[10]$	$\Omega_{cc} - \Xi_{cc} = 92(24)$	90-102
$\Omega_{bb}/\Xi_{bb} = 1.0049(13)$	$\Xi_{bb} = 9.94[6]$	$\Omega_{bb} - \Xi_{bb} = 49(13)$	60-73
$\Omega_{cc}^*/\Xi_{cc}^* = 1.0260(75)$	$\Xi_{cc}^* = 3.58^*$	$\Omega_{cc}^* - \Xi_{cc}^* = 94(27)$	91-100
$\Omega_{bb}^*/\Xi_{bb}^* = 1.0050(15)$	$\Xi_{bb}^* = 9.96^*$	$\Omega_{bb}^* - \Xi_{bb}^* = 50(15)$	60-72
$\Omega_{bc}/\Xi_{bc} = 1.0060(17)$	$\Xi_{bc} = 6.86[7]$	$\Omega_{bc} - \Xi_{bc} = 41(7)$	70-89

^{*} We have combined your results for the mass-splittings with the experimental value of $M_{\Xi_{cc}}$ and with the central value of $M_{\Xi_{bb}}$ in Eq. (1).

9. Conclusions

Our different results are summarized in Table 1 and agree in most cases with the potential model predictions given in [7, 11]:

- The mass-splittings between the spin 3/2 and 1/2 baryons, derived in Eqs. (14) and (15) are essentially due to the radiative corrections induced by the anomalous dimensions of the two-point correlator and seems to behave like $1/M_Q$.
- For the SU(3) mass-splittings, our results derived in Eq. (16) for the spin 1/2 and in Eq. (17) for the spin 3/2 indicate that the splittings due to the SU(3) breaking are almost independent on the spin of the heavy baryons and approximately behave like $1/M_Q$. These mass-behaviours can be qualitatively understood from the QCD expressions of the corresponding correlators where the leading mass corrections behave like m_s/m_Q .
- Finally, we obtain, in Eq. (19), the SU(3) mass-splittings between the $\Omega(bcs)$ and $\Xi(bcd)$, which is

about 1/2 of the potential model prediction.

Our previous predictions can be improved by including radiative corrections to the SU(3) breaking terms and can be tested, in the near future, at Tevatron and LHCb.

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